

Forward-Looking Measures of Higher-Order Dependencies with an Application to Portfolio Selection [†]

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Forward-Looking Measures of Higher-Order Dependencies with an Application to Portfolio Selection

Abstract

This paper provides implied measures of higher-order dependencies between assets. The measures exploit only forward-looking information from the options market and can be used to construct an implied estimator of the covariance, co-skewness, and co-kurtosis matrices of asset returns. We implement the estimator using a sample of US stocks. We show that the higher-order dependencies vary heavily over time and identify which driving them. Furthermore, we run a portfolio selection exercise and show that investors can benefit from the better out-of-sample performance of our estimator compared to various historical benchmark estimators. The benefit is up to seven percent per year.

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I Introduction

The dependence structure between assets is a key element of many problems in finance. It is needed, for example, to calculate the risk position of financial institutions, to measure contagion effects possibly leading to financial crises, to find appropriate hedging instruments, and to select optimal asset portfolios. Since the returns of many assets are not normally distributed, one has to go beyond covariances and take dependencies in higher-order moments, like co-skewness and co-kurtosis, into account to get a reliable picture of the dependence structure between assets.

However, estimating the dependence structure in higher-order moments is hard since the number of parameters to be estimated increases exponentially with the number of assets in the portfolio. Take, for example, a simple portfolio selection problem where the investor can choose among 30 stocks. If the investor ignores higher-order moments and adopts the classical mean-variance-approach, she has to estimate 'only' 495 parameters. However, if she incorporates skewness and kurtosis, the number of parameters to be estimated goes up to 46,375, most of them characterizing the dependence structure. This huge number of parameters is not only a high computational burden but also leads to serious estimation risk since the dependence between assets might change over time. For example, it is well known that correlations go up when the market goes down (see, e.g., Longin and Solnik (2001)).

We address this problem by suggesting a new way to estimate higher-order dependencies between assets. We impose a structure on the co-moment matrices to reduce the number of parameters and use option-implied information instead of time-series information to estimate the remaining ones. Thus, our approach is inherently forward-looking and incorporates most recent market information. Given the empirical evidence that implied estimators for the covariance matrix perform better than historical estimators (see, e.g., Kempf, Korn, and Saßning (2011)), one might expect that implied estimators for higher-order moments are a promising way to get useful estimates for the higher-order dependence structure of assets.

Our paper makes two major contributions. On the theoretical side, we develop the first implied dependence measures for higher-order moments. We capture higher-order dependencies by the skewness-correlation and the kurtosis-correlation. These correlations have intuitive interpretations. Our implied skewness-correlation (kurtosis-correlation) expresses the market expectation on how a shock in one asset will affect the volatility (skewness) of other assets. Furthermore, we show how these correlations can be used as the key element to construct an implied estimator of the full covariance, co-skewness, and co-kurtosis matrices. On the empirical side, we provide evidence on the characteristics of the implied correlations over time and identify factors driving the higher-order dependencies. Furthermore, we show that our implied estimator of higher-order co-moment matrices is valuable for investors. For a sample of US blue-chip stocks, we show that a portfolio strategy using our implied estimator beats several portfolio strategies using historical estimators. The monetary utility gains from using the implied estimator instead of a historical estimator are huge. They go up to seven percent per year. The investor benefits from our implied estimator the more, the more risk averse she is and the monetary utility gains are highest in a sub-period that contains the time of the financial crisis.

Our work is related to three strands of literature. The first strand consists of papers developing implied estimators of risk and dependence. Skintzi and Refenes (2005) propose an implied correlation index as a measure of average correlation in a market and Driessen, Maenhout, and Vilkov (2009) provide evidence on the difference between implied correlations and realized correlations. Buss and Vilkov (2012) use option-implied correlations to construct predictors of beta coefficients. Alternative option-implied betas are derived by Chang, Christoffersen, Jacobs, and Vainberg (2012) and Kempf, Korn, and Saßning (2011).¹ All these papers investigate dependence only in terms of second moments. We extend this literature by proposing implied dependence measures for higher-order moments.

The second strand of literature shows that option-implied information on higher-

¹See Baule, Korn, and Saßning (2013) for an empirical comparison of different implied beta estimators and Christoffersen, Jacobs, and Chang (2012) for a recent survey on implied estimation that also covers implied correlations and betas.

order moments can be valuable in portfolio problems. Kostakis, Panigirtzoglou, and Skiadopoulos (2011) and Aït-Sahalia and Brandt (2008) estimate whole marginal distributions from options data, i.e., they exploit implied information on all moments. However, the approach by Kostakis, Panigirtzoglou, and Skiadopoulos (2011) does not require any knowledge about dependence structures and Aït-Sahalia and Brandt (2008) use historical estimates to determine dependencies. DeMiguel, Plyakha, Upal, and Vilkov (2012) show that implied skewness can be used to improve the performance of parametric portfolio policies. However, they make no attempt to exploit higher-order co-moments. Thus, none of these papers on portfolio problems uses option-implied information on the higher-order dependence structure. We are the first to show that option-implied information on higher-order dependencies is useful for portfolio optimization.

Finally, we extend the scarce literature on estimating higher-order moments in the context of portfolio optimization. Harvey, Liechty, Liechty, and Muller (2010) use a Bayesian approach to account for the severe estimation risk via predictive distributions. Martellini and Ziemann (2010) develop structured estimators of higher-order co-moment matrices based on the assumptions of constant correlations or a single-factor model. However, none of these papers uses any option-implied information. This is the main difference to our paper which estimates higher-order moments using forward-looking information from the options market only.

The remainder of the paper is organized as follows. In Section II, we develop our implied dependence measures and the implied estimator of the full covariance, co-skewness, and co-kurtosis matrices. Section III describes the data and in Section IV we provide information about the empirical properties of the dependence measures and the factor underlying them. In section V, we present the portfolio application. Section VI concludes.

II Implied Estimators of Dependencies

Consider a set of N assets with random returns R_1, \dots, R_N and denote the centered returns as $\bar{R}_i := R_i - E(R_i), i = 1, \dots, N$. The n -th central return moment of asset i is denoted by $\mu_i^{(n)}$. To characterize the dependence between these assets, we define the following generalized correlation coefficients for second to fourth moments:

$$\begin{aligned}
\rho_{ij}^{Var} &:= \frac{E(\bar{R}_i \bar{R}_j)}{\sqrt{\mu_i^{(2)} \mu_j^{(2)}}}, \\
\rho_{1,ii}^{Skew} &:= \frac{E(\bar{R}_i^2 \bar{R}_j)}{\sqrt{\mu_i^{(4)} \mu_j^{(2)}}}, \\
\rho_{2,ijk}^{Skew} &:= \frac{E(\bar{R}_i \bar{R}_j \bar{R}_k)}{\sqrt{\mu_i^{(2)} \mu_j^{(4)} \mu_k^{(4)}}}, \\
\rho_{1,iii}^{Kurt} &:= \frac{E(\bar{R}_i^3 \bar{R}_j)}{\sqrt{\mu_i^{(6)} \mu_j^{(2)}}}, \\
\rho_{2,ii}^{Kurt} &:= \frac{E(\bar{R}_i^2 \bar{R}_j^2)}{\sqrt{\mu_i^{(4)} \mu_j^{(4)}}}, \\
\rho_{3,ijk}^{Kurt} &:= \frac{E(\bar{R}_i^2 \bar{R}_j \bar{R}_k)}{\sqrt{\mu_i^{(4)} \mu_j^{(4)} \mu_k^{(4)}}}, \\
\rho_{4,ijkl}^{Kurt} &:= \frac{E(\bar{R}_i \bar{R}_j \bar{R}_k \bar{R}_l)}{\sqrt{\sqrt{\mu_i^{(4)} \mu_j^{(4)}} \sqrt{\mu_k^{(4)} \mu_l^{(4)}}}},
\end{aligned} \tag{1}$$

with $i, j, k, l = 1, \dots, N$,

$i \neq j \neq k \neq l$.

ρ_{ij}^{Var} is the standard correlation coefficient. It measures the impact of a shock in one asset on the expected return in another asset. Consider a negative shock on asset j . Then a positive return correlation ρ_{ij}^{Var} implies that we expect a negative deviation from its mean return also for stock i . The other six correlations in Equations (1) have

similar interpretations. For example, if the skewness-correlation $\rho_{1,ij}^{Skew}$ is positive and we observe a negative shock in asset j , the conditional variance of asset i decreases. Similarly, a negative shock in asset j would lead to a lower conditional skewness of stock i if the kurtosis-correlation $\rho_{1,iiij}^{Kurt}$ is positive. Note that all correlation coefficients are bounded between -1 and $+1$.²

To reduce the number of parameters characterizing the dependence structure, we follow the same idea as Martellini and Ziemann (2010) and assume constant correlations. In particular, we assume that the dependence structure can be described using three correlation coefficients (ρ^{Var} , ρ^{Skew} , and ρ^{Kurt}) only. This implies that the standard correlations, the skewness-correlations and the kurtosis-correlations are all constant across assets and that the two skewness-correlations (four kurtosis-correlations) are equal.

We now estimate the correlations ρ^{Var} , ρ^{Skew} , and ρ^{Kurt} from a cross-section of options on individual stocks and the index option. The risk of the index is determined by the risk of the underlying stocks and the three correlations. We characterize the risk of the index portfolio using the higher-order co-moment matrices M_2 , M_3 , and M_4 introduced by Jondeau and Rockinger (2006):

$$\begin{aligned} M_2 &:= E\{(R - E\{R\})(R - E\{R\})^{tr}\}, \\ M_3 &:= E\{(R - E\{R\})(R - E\{R\})^{tr} \otimes (R - E\{R\})^{tr}\}, \\ M_4 &:= E\{(R - E\{R\})(R - E\{R\})^{tr} \otimes (R - E\{R\})^{tr} \otimes (R - E\{R\})^{tr}\}. \end{aligned} \tag{2}$$

M_2 is the covariance matrix, M_3 the co-skewness matrix, and M_4 the co-kurtosis matrix. R denotes the N -vector of asset returns and \otimes is the Kronecker product. Jondeau and Rockinger (2006) show that the variance $\mu_p^{(2)}$, skewness $\mu_p^{(3)}$, and

²This property can be seen using the Cauchy-Schwarz inequality: For two random variables X and Y , $|E(XY)| \leq \sqrt{E(X^2)E(Y^2)}$.

kurtosis $\mu_p^{(4)}$ of the index portfolio return can be written as

$$\begin{aligned}\mu_p^{(2)} &= \omega^{tr} M_2 \omega \\ \mu_p^{(3)} &= \omega^{tr} M_3 (\omega \otimes \omega) \\ \mu_p^{(4)} &= \omega^{tr} M_4 (\omega \otimes \omega \otimes \omega).\end{aligned}\tag{3}$$

The N -vector ω denotes the weights of the stocks in the index portfolio. Given our constant correlation assumption, we can rewrite Equations (3) as functions of the correlations ρ^{Var} , ρ^{Skew} , and ρ^{Kurt} . To do so, we define an auxiliary matrix $\Omega^{Var} \in M_{N \times N}(\mathbb{R})$ as

$$\begin{aligned}\Omega_{ii}^{Var} &= 0, \\ \Omega_{ij}^{Var} &= \sqrt{\mu_i^{(2)} \mu_j^{(2)}} \quad \text{for all } i \neq j.\end{aligned}$$

Using this auxiliary matrix Ω^{Var} we can rewrite the covariance matrix as

$$M_2 = \text{diag}(\mu_1^{(2)}, \dots, \mu_N^{(2)}) + \rho^{Var} \cdot \Omega^{Var}\tag{4}$$

and the portfolio variance as

$$\mu_p^{(2)} = \omega^{tr} [\text{diag}(\mu_1^{(2)}, \dots, \mu_N^{(2)}) + \rho^{Var} \cdot \Omega^{Var}] \omega.\tag{5}$$

In the same spirit, we define the auxiliary matrix $\Omega^{Skew} \in M_{N \times N^2}(\mathbb{R})$ as

$$\begin{aligned}\Omega_{iii}^{Skew} &= 0 \\ \Omega_{ijj}^{Skew} &= \sqrt{\mu_i^{(4)} \mu_j^{(2)}} \quad \text{for all } i \neq j \\ \Omega_{ijk}^{Skew} &= \sqrt{\mu_k^{(2)} \mu_i^{(4)} \mu_j^{(4)}} \quad \text{for all } i \neq j \neq k.\end{aligned}$$

The co-skewness matrix can be rewritten as

$$M_3 = \text{diag}(\mu_1^{(3)}, \dots, \mu_N^{(3)}) + \rho^{Skew} \cdot \Omega^{Skew} \quad (6)$$

and the portfolio skewness as

$$\mu_p^{(3)} = \omega^{tr} [\text{diag}(\mu_1^{(3)}, \dots, \mu_N^{(3)}) + \rho^{Skew} \cdot \Omega^{Skew}] (\omega \otimes \omega). \quad (7)$$

Finally, we define the auxiliary matrix $\Omega^{Kurt} \in M_{N \times N^3}(\mathbb{R})$ as

$$\begin{aligned} \Omega_{iiii}^{Kurt} &= 0 \\ \Omega_{iiij}^{Kurt} &= \sqrt{\mu_i^{(6)} \mu_j^{(2)}} && \text{for all } i \neq j \\ \Omega_{iijj}^{Kurt} &= \sqrt{\mu_i^{(4)} \mu_j^{(4)}} && \text{for all } i \neq j \\ \Omega_{iijk}^{Kurt} &= \sqrt{\mu_i^{(4)} \mu_j^{(4)} \mu_k^{(4)}} && \text{for all } i \neq j \neq k \\ \Omega_{ijkl}^{Kurt} &= \sqrt{\sqrt{\mu_i^{(4)} \mu_j^{(4)}} \sqrt{\mu_k^{(4)} \mu_l^{(4)}}} && \text{for all } i \neq j \neq k \neq l, \end{aligned}$$

leading to

$$M_4 = \text{diag}(\mu_1^{(4)}, \dots, \mu_N^{(4)}) + \rho^{Kurt} \cdot \Omega^{Kurt} \quad (8)$$

and

$$\mu_p^{(4)} = \omega^{tr} [\text{diag}(\mu_1^{(4)}, \dots, \mu_N^{(4)}) + \rho^{Kurt} \cdot \Omega^{Kurt}] (\omega \otimes \omega \otimes \omega). \quad (9)$$

Solving Equations (5), (7), and (9) for the generalized correlations leads to :

$$\rho^{Var} = \frac{\mu_p^{(2)} - \omega^{tr} [\text{diag}(\mu_1^{(2)}, \dots, \mu_N^{(2)})] \omega}{\omega^{tr} \Omega^{Var} \omega}, \quad (10)$$

$$\rho^{Skew} = \frac{\mu_p^{(3)} - \omega^{tr} [\text{diag}(\mu_1^{(3)}, \dots, \mu_N^{(3)})] (\omega \otimes \omega)}{\omega^{tr} \Omega^{Skew} (\omega \otimes \omega)}, \quad (11)$$

$$\rho^{Kurt} = \frac{\mu_p^{(4)} - \omega^{tr} [\text{diag}(\mu_1^{(4)}, \dots, \mu_N^{(4)})] (\omega \otimes \omega \otimes \omega)}{\omega^{tr} \Omega^{Kurt} (\omega \otimes \omega \otimes \omega)}. \quad (12)$$

The expressions on the right hand side of Equations (10) to (12) depend on the portfolio weights, the second to fourth central moments of the portfolio and the second, third, fourth, and sixth moments of the individual assets.

If options on the index and on all component stocks are available, we can estimate the correlation structure using option information only. We take the implied moments needed from plain-vanilla options written on the index and on individual assets and use the known index weights. With the estimation of the implied correlation estimators, we have also solved the problem of implied estimation of the whole co-moment matrices M_2 , M_3 , and M_4 in Equations (4), (6) and (8). All remaining parameters, in particular the auxiliary matrices Ω^{Var} , Ω^{Skew} , and Ω^{Kurt} , can easily be obtained from the implied moments of the individual stock returns.

III Data

The data set for our empirical study consists of the stocks constituting the Dow Jones Industrial Average (DJIA) for the period January 1998 to January 2012. For each point of time, we consider only the 30 stocks which form the index at that time.

To implement our implied estimators of generalized correlations and co-moment matrices, we need prices of European-style options for all individual stocks and the Dow Jones Index. We calculate these prices from the volatility surfaces provided by IvyDB. We use all available strike prices for the 30 days maturity bucket, select all out-of-the-money put and call options, and fit a cubic spline to obtain a smooth volatility curve for each stock and the index. Outside the available range of strike prices, we assume that the volatility curve is flat. Then, we select 1000 equally spaced strike prices on the interval $[0.003 \cdot S_i, 3 \cdot S_i]$, where S_i denotes the current spot price of the i th asset. For these 1000 strike prices we finally calculate prices of European options from the corresponding implied volatilities via the Black-Scholes formula. These calculations use the matching spot prices for all stocks and the index

as well as the risk-free interest rates provided by IvyDB. We calculate monthly option prices and choose the first trading day after the expiration day of options contracts at CBOE within a month, since there are liquid options with a time to maturity of about 30 days at these days.

Using this data set, we calculate model-free implied moments. This idea of not using a particular valuation model goes back to Breeden and Litzenberger (1978), who show that the complete risk-neutral return distribution can be derived from option prices if a continuum of strike prices is available. Based on the result by Bakshi and Madan (2000) that any payoff function can be spanned by explicit positions in options with different strike prices, Bakshi, Kapadia, and Madan (2003) provide pricing formulas for contracts whose payoffs equal the squared return, the cubed return, quadrupled return etc.³ For our purposes, we need the returns up to the power of six. The fair values of the corresponding contracts *Quad*, *Cubic*, *Quartic*, *Quintic*, and *Hexic* are provided in the appendix together with the formulas that show how the model-free implied second to sixth central return moments can be obtained from the prices of these contracts.

In Section IV we provide characteristics of the implied generalized correlations ρ^{Var} , ρ^{Skew} , and ρ^{Kurt} . In Section V, we compare a portfolio strategy using the implied estimator of the co-moment matrices M_2 , M_3 , and M_4 with portfolio strategies using various historical estimators. To implement the historical estimators and to calculate monthly out-of-sample returns for the trading strategies, we take stock prices (adjusted for dividends and stock splits) from Datastream. Since the historical estimators use estimation windows of up to 120 months, we have to take stock price data for the period January 1988 to January 2012.

³The formulas given in the original work by Bakshi, Kapadia, and Madan (2003) refer to log returns. See Christoffersen, Jacobs, and Chang (2012) for corresponding formulas referring to simple returns. The latter are used in this study.

IV Characteristics of Generalized Correlations

We now analyze the estimated dependence structure. In Section A we provide descriptive evidence on how the dependence structure changes over time and in Section B we identify factors that determine the strength of the dependencies.

A Dynamics of Generalized Correlations

Figure 1 shows the monthly implied estimates of the generalized correlation coefficients ρ^{Var} , ρ^{Skew} , and ρ^{Kurt} for the period February 1998 to January 2012.

[Insert Figure 1 about here]

The solid line in Figure 1 shows how the standard correlation ρ^{Var} evolves over time. Not surprisingly, it is always positive, i.e., a negative shock in one stock goes along with a price reduction in other stocks. On average, the correlation is 0.45 but it goes up to 0.85 during the recent financial crisis. This is consistent with earlier evidence that correlations go up when markets go down.

The sign of the skewness-correlation ρ^{Skew} is almost always negative, as shown by the dotted line. It can be as low as -0.49. This finding means that a negative shock in one of the assets is associated with an increase in the volatility of other stocks. This finding complements earlier evidence showing that a negative shock in a stock tends to increase the volatility of the same stock.

For the kurtosis-correlation ρ^{Kurt} , we generally find positive values (dashed line). They are almost as high as the value for ρ^{Var} and reach its maximum at 0.78. This result suggests that a negative shock in one stock makes the returns of other stocks more skewed to the left.

Figure 1 shows that all implied dependencies change dramatically over time. This finding suggests that not only the standard correlation but also the skewness-correlation and the kurtosis-correlation are hard to estimate from time-series data.

From 2004 to 2007, the (absolute) values of all three dependence measures are relatively low, and from 2008 onwards they are relatively high. This is an indication for stronger contagion effects in the financial crisis. A shock in one stock affects the moments of other stocks more severely during the crisis than during quiet periods.

We also observe that the three implied dependence measures clearly move together. If ρ^{Var} is high, then ρ^{Skew} tends to be low (more negative) and ρ^{Kurt} tends to be high. The corresponding correlations between ρ^{Var} and ρ^{Skew} and between ρ^{Var} and ρ^{Kurt} are -0.73 and 0.96, respectively. This is bad news for investors since a negative shock in one stock has a strong negative impact on the expected returns, the variances and the skewnesses of other stocks. The expected returns decrease, the variances increase, and the stocks become more skewed to the left. Thus, the saying that diversification benefits tend to be low at times when they are most needed holds not only for second moments but also for moments of higher order.

B Determinants of Generalized Correlations

We now analyze factors determining the generalized correlation coefficients. We run regressions with our generalized correlation coefficients ρ^{Var} , ρ^{Skew} , and ρ^{Kurt} as the dependent variables.

Our first explanatory variable is the market risk since it has been documented for a long time that the standard correlation goes up when market risk goes up (see, e.g., King and Wadhwani (1990), Longin and Solnik (1995)). We use two variables to capture market risk. The first variable, the index variance, measures the general market risk and the second variable, the index skewness, captures the crash risk in the market. Both variables are calculated for the same trading days as our correlation coefficients using our model in Section II.

Longin and Solnik (2001) show that the standard correlation is related to the market trend. Therefore, we include the market trend as an additional variable in our regressions. More specifically, we define a dummy variable which takes on the value

1 when the return of the previous month is negative and larger (in absolute terms) than one standard deviation. Otherwise this downturn dummy takes the value of zero.

Next, we consider investor sentiment in our regressions since Kumar and Lee (2006) have shown that sentiment makes retail investors trade similarly leading to return comovements. To capture retail investor sentiment, we use the Individual Investor Sentiment Index (AAII) which is obtained from a survey of the American Association of Individual Investors among its members.

As a further explanatory variable we use the importance of common factors for explaining stock returns. The rationale is that we expect to see a higher correlation when stock returns depend on common factors to a higher degree. To capture the relative importance of common factors, we estimate a Carhart 4-factor model using daily returns of the previous month. We then calculate how much of the return variance is explained by this model and relate the explained variance to the overall variance. We do so for each stock in our sample separately, calculate the average across stocks, and take this average number as our measure of the importance of common factors for explaining stock returns.

Since our generalized correlation measures are derived from options prices and, therefore, reflect the expectations of market participants in the options market, we use two further control variables. We control for the impact of the general economic outlook on the expectations by using the OECD Composite Leading Indicator US as an additional explanatory variable. The Composite Leading Indicator comprises different macroeconomic and financial variables that are known to be leading indicators of economic growth. In addition, we take into account that market participants might change their expectations about return dependencies only gradually. Therefore, we also include the corresponding lagged (generalized) correlation in the regressions.

[Insert Table 1 about here]

In Table 1 we present the results of our regressions. In all regressions, we calculate

Newey-West standard errors with 12 lags to account for residual autocorrelation and heteroscedasticity. The dependent variable is the implied standard correlation, ρ^{Var} , in the first column, the implied skewness-correlation, ρ^{Skew} , in the second column, and the implied kurtosis-correlation, ρ^{Kurt} , in the third column.

The first column shows that the standard correlation increases when the market risk becomes larger. The positive coefficient for the index variance means that the correlation goes up when the market becomes more volatile, and the negative coefficient for the index skewness implies that the correlation goes up when the market becomes more skewed to the left. Thus, both, the general market risk (index variance) and the crash risk (index skewness), have a significant impact on the standard correlation. This is consistent with what we expect given the evidence in the literature. Moreover, we find a significant negative impact of the downturn dummy, which means that correlation goes up when the market goes down. This is consistent with the view that the market trend has an additional influence even after controlling for market risk. The highly significant and negative coefficient of the investor sentiment proxy indicates that the standard correlation goes up when investor sentiment becomes bad. This finding is consistent with Kumar and Lee (2006) and suggests that investors treat different stocks similarly when their sentiment is bad. There is also a strong impact of the importance of common factors for explaining stock returns. This finding is highly sensible: If stock returns are mainly driven by common factors and not by idiosyncratic factors, we observe a higher standard correlation between the stocks. We also find that the correlation expected by the market participants depends on what the market participants expect about the future economic growth. The worse the economic outlook, the higher the implied correlation is. This is consistent with our findings that the correlation goes up when the expected market risk (captured by the implied variance and implied skewness) increases: The worse the expectation about the future situation, the higher the correlation is. The significantly positive coefficient for the lagged correlation suggests that the correlation expectations of market participants are somewhat persistent.

Turning to the higher-order correlations shows that the kurtosis-correlation depends

on the factors in the same way as the standard correlation. Thus, if a factor increases the standard correlation it also increases the kurtosis-correlation. In such a situation, investors who observe a negative shock in one stock not only expect that the returns of other stocks go down (reflected in the positive standard correlation) but they also expect that the crash risk of other stocks goes up (reflected in the positive kurtosis-correlation).

Looking at the skewness-correlation shows that ρ^{Skew} also depends on the market risk. The signs of the coefficients suggest that an increase in the market risk (index becomes more volatile and more skewed to the left) makes the skewness-correlation more negative. However, we find only a significant impact of the crash risk (measured by the index skewness) whereas the general market risk is not significant at conventional levels (p-value = 0.17). Besides market risk, the importance of common factors for explaining stock returns has a strong impact on the skewness-correlation: The skewness-correlation becomes the more negative, the more important common factor are for explaining stock returns. The downturn dummy and the investor sentiment have no significant impact on the skewness-correlation. Like for the other implied correlation measures, we find that the expected skewness-correlation depends on the general economic outlook and on the expectation a month ago: The worse the outlook, the more negative the skewness-correlation is.

Overall, the standard correlation and higher-order correlations are well explained by the explanatory variables. The explanatory power is equally high in all regressions. Furthermore, we find that the control variables economic outlook and lagged correlation are significant, no matter which implied correlation we analyze. This suggests that the investors adjust their expectations about future dependencies only gradually and in accordance with their expectations about future economic growth.

Looking at the other explanatory variables shows that standard correlation and kurtosis-correlation are driven by the same set of variables. First, they increase with the market risk, they are higher in market downturns, during periods when investor sentiment is bad and when stock returns are better explained by common

factors. When looking at the skewness-correlation, only a subset of our explanatory variables (market risk, importance of common factors) is significant whereas the market downturn and investor sentiment are not significant at the conventional levels. To understand that difference, we have to keep in mind that the standard correlation and the kurtosis-correlation provide directional information about future returns whereas the skewness-correlation provides only information about the variability of future returns. The standard correlation (kurtosis-correlation) tells us what return (crash risk) the investors expect in other stocks when they receive a negative signal in one stock. For example, it is highly sensible that a negative signal in one stock makes them expect a worse return (higher crash risk) in other stocks when their sentiment is bad. However, it is not clear why that signal would make them expect the other stocks to be more volatile when their sentiment is bad. That is what a significant impact of investor sentiment on skewness-correlation would imply.

V Portfolio Application

A The Portfolio Problem

We analyze a standard one-period expected utility maximization. For an infinitely differentiable utility function U , the utility of the investor's terminal wealth can be written as:

$$U(W) = \sum_{k=0}^{\infty} \left[\frac{U^{(k)}(E\{W\})}{k!} (W - E\{W\})^k \right], \quad (13)$$

with $W = (1 + \omega^{tr} R)$.

ω denotes a column vector of length N and contains the portfolio weights of the N different assets. R denotes the corresponding column random vector of asset returns over the period. Without loss of generality, the investor's initial wealth is

normalized to unity in Equation (13). We follow the typical approach and assume that the utility function is well approximated by a fourth-order polynomial. Thus, the expected utility of the investor is given as:

$$E\{U(W)\} \approx U(E\{W\}) + \frac{U^{(2)}(E\{W\})}{2} \mu^{(2)} + \frac{U^{(3)}(E\{W\})}{6} \mu^{(3)} + \frac{U^{(4)}(E\{W\})}{24} \mu^{(4)}, \quad (14)$$

$$\begin{aligned} \text{with } \mu^{(2)} &= \omega^{tr} M_2 \omega, \\ \mu^{(3)} &= \omega^{tr} M_3 (\omega \otimes \omega), \\ \mu^{(4)} &= \omega^{tr} M_4 (\omega \otimes \omega \otimes \omega). \end{aligned}$$

M_2 denotes the covariance matrix of asset returns, M_3 the co-skewness matrix, and M_4 the co-kurtosis matrix. It is well known that expected returns are very difficult to estimate (see, e.g., Merton (1980)) and that portfolio strategies ignoring expected returns typically perform better (see, e.g., Michaud (1989), Best and Grauer (1991), and Chopra and Ziemba (1993)). Therefore, we make no attempt to estimate expected returns and focus on minimizing the risk of the portfolio. The portfolio risk depends on the variance, skewness, and kurtosis of the portfolio return.

To be able to solve for the optimal portfolio weights, we have to specify the utility function in Equation (14). We assume that the investor has CRRA preferences with a relative risk aversion γ . We impose short-sales constraints since such restrictions typically improve the out-of-sample performance of investment strategies (see, e.g., Frost and Savarino (1988), Jagannathan and Ma (2003), and DeMiguel, Garlappi, and Uppal (2009)). Given these assumptions, the optimization problem

of the investor can be written as:

$$\arg \max_{\omega \in \mathbb{R}^N} \left[-\frac{\gamma}{2} \omega^{tr} M_2 \omega + \frac{\gamma(\gamma+1)}{6} \omega^{tr} M_3 (\omega \otimes \omega) - \frac{\gamma(\gamma+1)(\gamma+2)}{24} \omega^{tr} M_4 (\omega \otimes \omega \otimes \omega) \right], \quad (15)$$

$$\begin{aligned} \text{s.t. } & \sum_{i=1}^N \omega_i = 1, \\ & \omega_i \geq 0, \forall i. \end{aligned}$$

Equation (15) shows that an investor with CRRA utility has a preference for low variance, high skewness, and low kurtosis. This preference structure is consistent with Rubinstein (1973), Kraus and Litzenberger (1976), and Scott and Horvath (1980) who show that only weak assumptions on the utility functions are needed to derive preferences for low variance, high skewness, and low kurtosis. Furthermore, Equation (15) shows that higher moments are the more important to an investor, the more risk averse she is.

B Design of the Empirical Portfolio Study

To implement the optimal investment strategy arising from (15), we have to estimate the matrices M_2 , M_3 , and M_4 . We use five different ways to estimate the matrices. The first estimator is our fully-implied estimator derived in Section II. The other four estimators serve as historical benchmarks. We use the simple sample estimator (Sample) as our first benchmark. The other benchmarks are the estimators derived by Martellini and Ziemann (2010) for estimating higher-order moments. Their two structured estimators assume constant correlations (CC) and a single-factor model (FM), respectively. Their other two estimators shrink the sample estimates of the moment matrices M_2 , M_3 , and M_4 towards the estimates obtained under the constant correlation (Sh_CC) or the single-factor model (Sh_FM).

Based on each estimator, we set up the following investment strategy. In each month, we use the respective estimator to obtain M_2 , M_3 , and M_4 . Based on these estimates, we derive the optimal portfolio using (15). Then we calculate the out-of-sample one-month return of this portfolio. This procedure gives us 168 monthly portfolio returns for each of the estimators.

To compare the performance of the investment strategy based on the implied estimator with the benchmark strategies, we calculate monetary utility gains (MUGs) as in Ang and Bekaert (2002). For $\gamma \neq 1$ the MUG is given as:

$$\frac{1}{168} \sum_{t=1}^{168} \frac{(1 + r_t^{impl})^{1-\gamma} - 1}{1 - \gamma} = \frac{1}{168} \sum_{t=1}^{168} \frac{((1 + MUG) \cdot (1 + r_t^{bm}))^{1-\gamma} - 1}{1 - \gamma}. \quad (16)$$

r^{impl} denotes the return of an trading strategy using the implied estimator and r^{bm} the return of a benchmark strategy using a historical estimator. Thus, MUG is the monetary compensation (in percentage points) that an investor requires to be willing to switch from the portfolio strategy using the implied estimator to a benchmark portfolio strategy using a historical estimator. A positive MUG means that the investor prefers the implied estimator and is willing to use the historical estimators only if she gets a compensation. Therefore, a positive MUG indicates that the implied estimator is superior to the respective historical estimator. In the following section we report annualized MUGs which are calculated as $(1 + MUG)^{12} - 1$.

C Main Results

Table 2 reports the annualized monetary utility gains (MUGs) of the implied portfolio strategy relative to the five historical benchmark strategies based on the sample estimator (Sample), the estimator based on the assumption of constant correlations (CC), the estimator using a factor model (FM), the shrinkage model towards constant correlation (Sh_CC), and shrinkage model towards the factor model (Sh_FM), respectively. The relative risk aversion of the investor is $\gamma = 10$. In the first column

the historical estimators use an estimation window of 60 months and in the second column an estimation window of 120 months.

[Insert Table 2 about here]

The main result of Table 2 is that the MUGs are positive in all cases (no matter whether the historical estimators use an estimation window of 60 or 120 months). This means that investors would be willing to use a historical estimator instead of the implied estimator only if they get a monetary compensation. The size of the required compensation ranges from 2.4% to 4.8% per year. These are huge numbers given that the average returns of the benchmark strategies are only between six and seven percent per year. This result shows that our implied estimator is very valuable for investors.

In Table 3 we compare the implied strategy with various partially implied strategies to analyze how much of the MUGs come from using option-implied information to estimate the co-skewness and co-kurtosis matrices. The partially implied strategies use our implied estimator for the covariance matrix and historical estimators for the higher moments. The historical estimators are the same as in Table 2.

[Insert Table 3 about here]

Table 3 shows that the MUGs are much smaller than in Table 2 but still positive. The required compensation is between 0.4% and 1.2% per year. This means that investors would be willing to use a historical estimator for higher moments and co-moments instead of the implied estimator only if they get a sizable compensation. The level of required compensation is about the same as in Ang and Bekaert (2002). This suggests that using implied estimators for higher moments is about as important for investors as taking into account that model parameters might be different in different regimes. This is sensible since a major advantage of our implied estimator is that it does not use historical information and, thus, immediately adjusts when the regime changes. Comparing our results with those in Martellini and Ziemann (2010) shows

that the marginal contribution of our implied estimator for higher-order moments is larger than the marginal contribution of their structured estimators. They report a maximum required compensation of 0.26% per year in Panel B of their Table 10. In Table 4 we provide information about the portfolio composition when using different estimators. We report the average number of stocks held in the portfolio (first column) and the average Gini coefficient of the portfolio (second column). The averages are calculated across the monthly portfolio weight observations.

[Insert Table 4 about here]

Table 4 shows that the implied estimator leads to more concentrated portfolios. The number of stocks held is lower and the Gini coefficient is higher. On average, the investor picks only about nine stocks from the investment universe of thirty stocks when using the implied estimator. For the historical estimators, the number of stocks held is larger by two to four stocks. Together with our results in Table 2, this finding suggests that the implied estimators allows the investors to better pick appropriate stocks.

D Robustness Checks

In Table 5 we repeat the analysis of Table 2 but now for different levels of risk aversion. The risk aversion is varied from $\gamma = 5$ to $\gamma = 15$. We use an estimation window of 60 months for the benchmark strategies.

[Insert Table 5 about here]

Table 5 clearly shows that the MUGs are the larger the more risk averse the investors are. This finding holds for each benchmark strategy. Averaged across the benchmark strategies, we find an average MUG of 2.2% for $\gamma = 5$, 3.2% for $\gamma = 10$, and 4.1% for $\gamma = 15$. Thus, investors value the implied estimator more if they are more risk averse. This result is highly sensible since variance, skewness, and kurtosis

are the more important, the more risk averse investors are (see Equation (15)). Therefore, investors with a higher risk aversion benefit more from an improved moment estimation.

We next check whether our results depend on the size of the investment universe. So far, we assume that the investor can choose from an investment universe of 30 stocks. We now restrict the investment universe to 20 (10) stocks that are randomly drawn out of the 30 stocks constituting the Dow Jones Industrial Average (DJIA). Using this restricted investment universe, we repeat the analysis of Table 2. We do so 100 times for the investment universe of 20 stocks and another 100 times for the investment universe of 10 stocks. The average MUGs and the percentage of positive MUGs (in brackets) are reported in Table 6. For comparison reasons we repeat the numbers for $N=30$ stocks from Table 2. We again use an estimation window of 60 months for the benchmark strategies.

[Insert Table 6 about here]

Table 6 shows that the average MUGs are positive in all cases. Thus, our main result is robust with respect to the size of the investment universe. Averaged across the benchmark strategies, we find a positive relation between the size of the investment universe and the size of the MUGs (average MUG = 2.52% for $N=10$, average MUG = 2.61% for $N=20$, MUG = 3.21% for $N=30$), but this relation does not hold when looking at the benchmark strategies separately. However, what we see for each benchmark strategy is that the probability of achieving a positive MUG goes up when the investment universe becomes larger. Averaged across the benchmark strategies, we find that the probability of achieving a positive MUG is 81.2% for $N=10$ and 91.6% for $N=20$. This suggests that an investor can pocket the gains of our implied estimator more easily when she holds a larger portfolio.

In our final robustness check we split our sample in two sub-samples of equal length. The first sub-sample covers the period from February 1998 to January 2005 and the second sub-sample the period from February 2005 to January 2012. We repeat the

analysis of Table 2 for both sub-samples and again use an estimation window of 60 months for the benchmark strategies. Table 7 presents the resulting MUGs.

[Insert Table 7 about here]

Table 7 suggests that our main result is not sample specific. The MUGs are positive in both sub-samples. Interestingly, the MUGs are much larger in the second sub-period. A possible reason for this finding is that this period covers the financial crisis. Kempf, Korn, and Saßning (2011) argue that implied estimators perform particularly well in crisis periods for two reasons: First, historical time series are less useful in crisis periods due to the strong inflow of new information. Second, option prices carry more information in crisis periods since the fraction of informed traders in the option market goes up. This makes option prices more informative. Therefore, the implied estimator is particularly attractive in crisis periods which should lead to higher MUGs. That is exactly what we find.

VI Conclusions

This paper investigates higher-order dependencies between assets. Since dependencies are known to change over time and, therefore, hard to estimate from time-series information, we suggest a novel way to estimate higher-order dependencies. Our model allows us to derive generalized correlation coefficients for skewness and kurtosis using only current option-implied information. We do not need any time-series information to estimate them. Thus, our approach is inherently forward-looking and incorporates most recent information from options markets. The implied generalized correlations have intuitive interpretations in terms of the expected impact that a shock in one asset has on the expected return, variance, and skewness of other assets. The correlations build the basis for an implied estimator of the full covariance, co-skewness, and co-kurtosis matrices that we also present in this paper.

In an empirical study for US blue-chip stocks, we provide evidence on the characteristics of the implied generalized correlations over time. We document that the

correlations vary heavily over time and detect the factors that determine the implied correlations. We first show that investors adjust their expectations about future dependencies only gradually and in accordance with their expectations about future economic growth. We then find that standard correlation and kurtosis-correlation are driven by the same set of factors. Both correlations increase with the market risk, they are higher in market downturns, during periods when investor sentiment is bad and when stock returns are better explained by common factors. The skewness-correlation, however, depends only on a subset of our explanatory variables (market risk, importance of common factors). This is sensible since the standard correlation and the kurtosis-correlation provide directional information about future returns whereas the skewness-correlation provides only information about the variability of future returns.

We use the implied generalized correlations in a empirical portfolio optimization exercise. An out-of-sample study for the same universe of US blue-chip stocks shows that our implied estimator of higher-order moment matrices is very valuable for investors that seek optimal portfolios based on second to fourth moments. We find that a portfolio strategy based on the implied estimator beats several benchmark strategies based on historical estimators. The monetary utility gains from using the implied estimator instead of historical estimators are huge and can reach up to seven percent per year. In a robustness analysis, we find that the implied estimator is superior for a wide range of investors with different risk aversions, for alternative sizes of the investment universe, and different sub-periods.

A major issue for future research is the risk-adjustment of option-implied higher moments. The moments used in this study are obtained under the risk-neutral measure, whereas most applications require moments under the physical measure. If appropriate risk-adjustments are available, further improvements in the assessment of higher-order dependencies are likely. Unfortunately, such a risk-adjustment is difficult since very little is known about risk premiums for higher-order co-moments. The implied estimators of these co-moments that we develop in this paper, however, offer a way to study such risk premiums empirically.

Appendix

Bakshi, Kapadia, and Madan (2003) show how to price contracts whose payoffs equal different powers of the returns. For our analysis, we need powers up to the order of six. Denote the arbitrage-free prices at time t for contracts maturing at time $t + \tau$ by *Quad* (squared returns), *Cubic* (cubed returns), *Quartic* (quadrupled returns), *Quintic* (returns to the power of five), and *Hexic* (returns to the power of six). According to Bakshi, Kapadia, and Madan (2003), these prices equal

$$\begin{aligned}
 Quad &= \frac{2}{S_t^2} \left[\int_{S_t}^{\infty} C(t, \tau, K) dK + \int_0^{S_t} P(t, \tau, K) dK \right] \\
 Cubic &= \frac{6}{S_t^2} \left[\int_{S_t}^{\infty} \frac{K - S_t}{S_t} C(t, \tau, K) dK + \int_0^{S_t} \frac{K - S_t}{S_t} P(t, \tau, K) dK \right] \\
 Quartic &= \frac{12}{S_t^2} \left[\int_{S_t}^{\infty} \left(\frac{K - S_t}{S_t} \right)^2 C(t, \tau, K) dK + \int_0^{S_t} \left(\frac{K - S_t}{S_t} \right)^2 P(t, \tau, K) dK \right] \\
 Quintic &= \frac{20}{S_t^2} \left[\int_{S_t}^{\infty} \left(\frac{K - S_t}{S_t} \right)^3 C(t, \tau, K) dK + \int_0^{S_t} \left(\frac{K - S_t}{S_t} \right)^3 P(t, \tau, K) dK \right] \\
 Hexic &= \frac{30}{S_t^2} \left[\int_{S_t}^{\infty} \left(\frac{K - S_t}{S_t} \right)^4 C(t, \tau, K) dK + \int_0^{S_t} \left(\frac{K - S_t}{S_t} \right)^4 P(t, \tau, K) dK \right],
 \end{aligned}$$

where S_t is the current spot price, K the strike price, and C and P denote the prices of call and put options, respectively. Using these contract prices, the model-free implied second, third, fourth, and sixth central return moments of asset i can be written as

$$\begin{aligned}
\mu_{i,impl}^{(2)} &= e^{r\tau} Quad_i - E^q[R_i]^2, \\
\mu_{i,impl}^{(3)} &= e^{r\tau} Cubic_i - 3E^q[R_i]e^{r\tau} Quad_i + 2E^q[R_i]^3, \\
\mu_{i,impl}^{(4)} &= e^{r\tau} Quartic_i - 4E^q[R_i]e^{r\tau} Cubic_i + 6E^q[R_i]^2e^{r\tau} Quad_i - 3E^q[R_i]^4, \\
\mu_{i,impl}^{(6)} &= e^{r\tau} Hexic_i - 6e^{r\tau} Quintic_i E^q[R_i] + 15e^{r\tau} Quartic_i E^q[R_i]^2 \\
&\quad - 20e^{r\tau} Cubic_i E^q[R_i]^3 + 15e^{r\tau} Quad_i E^q[R_i]^4 - 5E^q[R_i]^6,
\end{aligned} \tag{17}$$

where r denotes the risk-free interest rate per year for the period τ and $E^q[R_i]$ the risk-neutral expectation of the i th asset. The risk-neutral expectation is given by

$$E^q[R_i] = e^{r\tau} - 1.$$

The expressions in Equations (17) together with this expectation deliver the moments of individual stocks that we use for our analysis. Implied moments for the Dow Jones Index are obtained in the same way.

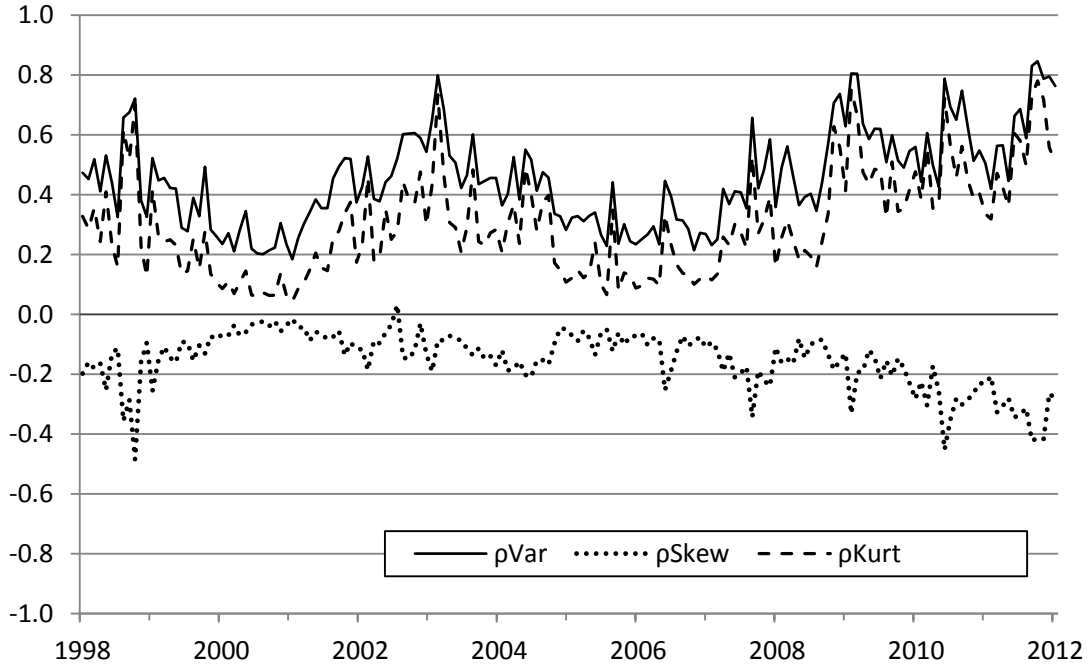
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Figure 1: Implied Estimates of Generalized Correlations Over Time.



This figure shows the implied estimates of the standard and the higher-order correlations ρ^{Var} , ρ^{Skew} , and ρ^{Kurt} over time. All correlations are calculated as described in Section II. The time period is February 1998 to January 2012 and the stock universe consists of the 30 stocks included in the Dow Jones Industrial Average (DJIA).

Table 1: Determinants of Implied Correlations.

	ρ^{Var}	ρ^{Skew}	ρ^{Kurt}
Constant	0.940 (0.004)	-0.613 (0.000)	1.982 (0.000)
Index Variance	6.801 (0.067)	-4.522 (0.172)	8.881 (0.050)
Index Skewness	-0.066 (0.002)	0.153 (0.000)	-0.143 (0.000)
Market Downturn	0.059 (0.005)	-0.019 (0.124)	0.057 (0.087)
Investor Sentiment	-0.063 (0.000)	0.000 (0.995)	-0.070 (0.009)
Importance of Common Factors	0.287 (0.000)	-0.161 (0.000)	0.267 (0.000)
Economic Outlook	-0.010 (0.001)	0.008 (0.000)	-0.021 (0.000)
Lagged Correlation	0.500 (0.000)	0.359 (0.000)	0.386 (0.000)
Adjusted R^2	0.745	0.745	0.657

This table shows the results of multivariate regressions using monthly observations for the period February 1998 to January 2012. The dependent variable is the standard correlation, ρ^{Var} , in the first column, the skewness-correlation, ρ^{Skew} , in the second column, and the kurtosis-correlation, ρ^{Kurt} , in the third column. The time series of the dependent variables are shown in Figure 1. The independent variables are as follows. We use the implied index variance and the implied index skewness to capture symmetric market risk and market crash risk, respectively. Both variables are calculated using our model in Section II. To measure the market trend, we use a downturn dummy that takes on the value 1 when the return of the previous month is negative and larger (in absolute terms) than one standard deviation. Otherwise the downturn dummy takes the value of zero. We capture investor sentiment using the Individual Investor Sentiment Index of the American Association of Individual Investors. The importance of common factors measures how important common factors are for explaining stock returns. To capture the importance of common factors, we first estimate for each stock a Carhart 4-factor model using daily returns of the previous month. We then calculate how much of the return variance is explained by this model and relate the explained variance to the overall variance. We do so for each stock in our sample, calculate the average across stocks, and take this average number as our explanatory variable. The economic outlook is measured by the OECD Composite Leading Indicator US. Finally, we use the corresponding lagged correlations to capture the effect that market participants might revise their expectations only gradually. In all regressions, we calculate Newey-West standard errors with 12 lags and report p-values in brackets.

Table 2: Monetary Utility Gains: Implied Moment Estimator versus Historical Estimators.

	60 Months	120 Months
Sample	2.66%	2.69%
CC	4.82%	4.11%
FM	2.82%	2.71%
Sh_CC	3.35%	3.32%
Sh_FM	2.39%	2.69%

This table shows the annualized monetary utility gains of the investment strategy using the implied estimator relative to an investment strategy using a historical estimator. The monetary utility gains are calculated based on Equation (16) and annualized as $(1 + MUG)^{12} - 1$. The historical estimators are the sample estimator (Sample), the estimator based on the assumption of constant correlations (CC), the estimator using a one-factor model (FM), the shrinkage model towards constant correlation (Sh_CC), and the shrinkage model towards the factor model (Sh_FM). In the first column the historical estimators use an estimation window of 60 months and in the second column an estimation window of 120 months. The relative risk aversion of the investor is $\gamma = 10$. The out-of-sample sample period is February 1998 to January 2012 and the investment universe consists of the stocks included in the Dow Jones Industrial Average (DJIA).

Table 3: Monetary Utility Gains: Implied Moment Estimator versus Partially Implied Estimators.

	60 Months	120 Months
Sample	1.25%	0.37%
CC	0.66%	0.71%
FM	0.49%	0.42%
Sh_CC	0.63%	0.68%
Sh_FM	0.82%	0.69%

This table shows the annualized monetary utility gains of the investment strategy using the implied estimator relative to an investment strategy using a partially implied estimator. The monetary utility gains are calculated based on Equation (16) and annualized as $(1 + MUG)^{12} - 1$. The partially implied estimators consist of an implied estimator for the covariance matrix and historical estimators for the co-skewness and co-kurtosis matrices. The historical estimators are the sample estimator (Sample), the estimator based on the assumption of constant correlations (CC), the estimator using a one-factor model (FM), the shrinkage model towards constant correlation (Sh_CC), and the shrinkage model towards the factor model (Sh_FM). In the first column the historical estimators use an estimation window of 60 months and in the second column an estimation window of 120 months. The relative risk aversion of the investor is $\gamma = 10$. The out-of-sample sample period is February 1998 to January 2012 and the investment universe consists of the stocks included in the Dow Jones Industrial Average (DJIA).

Table 4: Concentration Measures of Portfolios Based on Different Moment Estimators.

	Number of Stocks Held	Gini Coefficient
Implied	8.77	82.59%
Sample	11.13	78.02%
CC	12.82	78.05%
FM	12.12	77.60%
Sh_CC	12.97	77.07%
Sh_FM	12.77	76.03%

This table shows the average number of stocks and the Gini coefficient of the portfolios underlying Table 2. We report values for the portfolios using the implied estimator (Implied), the sample estimator (Sample), the estimator based on the assumption of constant correlations (CC), the estimator using a one-factor model (FM), the shrinkage model towards constant correlation (Sh_CC), and the shrinkage model towards the factor model (Sh_FM). The estimation window for the historical estimators is 60 months.

Table 5: Monetary Utility Gains for Different Levels of Risk Aversion: Implied Moment Estimator versus Historical Estimators.

	$\gamma = 5$	$\gamma = 10$	$\gamma = 15$
Sample	2.05%	2.66%	2.79%
CC	3.09%	4.82%	7.00%
FM	2.00%	2.82%	3.61%
Sh_CC	2.31%	3.35%	4.61%
Sh_FM	1.78%	2.39%	2.71%

This table replicates Table 2 for different levels of relative risk aversion. The relative risk aversion of the investor is $\gamma = 5$, $\gamma = 10$, and $\gamma = 15$, respectively. The estimation window for the historical estimators is 60 months.

Table 6: Monetary Utility Gains for Different Sizes of the Investment Universe:
Implied Moment Estimator versus Historical Estimators.

	N = 10		N = 20		N = 30
Sample	2.43%	(79%)	2.08%	(85%)	2.66%
CC	3.45%	(89%)	4.46%	(100%)	4.82%
FM	1.85%	(73%)	1.70%	(88%)	2.82%
Sh_CC	2.84%	(86%)	3.01%	(99%)	3.35%
Sh_FM	2.02%	(79%)	1.81%	(86%)	2.39%

This table replicates Table 2 for different sizes of the investment universe. We restrict the investment universe to N=20 (N=10) stocks which are randomly drawn out of the 30 stocks constituting the Dow Jones Industrial Average (DJIA). Using this restricted investment universe, we repeat the analysis of Table 2. We do so 100 times for the investment universe of 20 stocks and another 100 times for the investment universe of 10 stocks. The average MUGs and the percentage of positive MUGs (in brackets) are reported for N=10 and N=20. For comparison reasons we repeat the numbers for N=30 stocks from Table 2. The estimation window for the historical estimators is 60 months.

Table 7: Monetary Utility Gains for Different Sub-periods: Implied Moment Estimator versus Historical Estimators.

	Period Feb 1998 – Jan 2005	Period Feb 2005 – Jan 2012
Sample	1.31%	4.16%
CC	4.37%	5.32%
FM	1.95%	3.79%
Sh_CC	2.43%	4.38%
Sh_FM	1.25%	3.66%

This table replicates Table 2 for different sub-periods. The first sub-period covers February 1998 to January 2005 and the second sub-period February 2005 to January 2012. The estimation window for the historical estimators is 60 months.